Getting Your "Rep"s for a Great Algebra Workout

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It's about algebraic thinking, not rote use of algorithms

From the *Principles and Standards for School Mathematics (PSSM):*

- "The Algebra standard emphasizes relationships among quantities, including functions, ways of representing mathematical relationships, and the analysis of change"
- "But algebra is more than moving symbols around"

_____. *Principles and Standards for School Mathematics.* National Council of Teachers of Mathematics (NCTM): Reston, VA. 2000.

But it will not be easy . . .

"The research has made clear that teaching and learning with multiple representations linked to one another and to problem situations is necessarily very complex."

Greenes, Carole E. and Rheta Rubinstein, editors. "A History of Algebra in the School Curriculum," Algebra and Algebraic Thinking in School Mathematics, Seventieth Yearbook. National Council of Teachers of Mathematics (NCTM): Reston, VA, 2008.

Students Need to Think About Situations

In planning a field trip, how many buses are needed for all of the Algebra classes?

How do we create a formula to convert temperatures from Celsius to Fahrenheit?

Not Magic or a Bag of Tricks

"algebra should not be taught as a collection of tricks . . . Students should see algebra as an aid for thinking rather than a bag of tricks"

Moses, Barbara, editor. Algebraic Thinking Grades K-12. National Council of Teachers of Mathematics (NCTM): Reston, VA, 1999.
Thorpe, John A. "Algebra: What Should We Teach and How Should We Teach It?," *Research Issues in the Learning and Teaching of Algebra*, 1989.

Representation: Translating Words to Expressions

How do students approach a problem like this:

Six times as many students as teachers

Problem solving in meaningful contexts

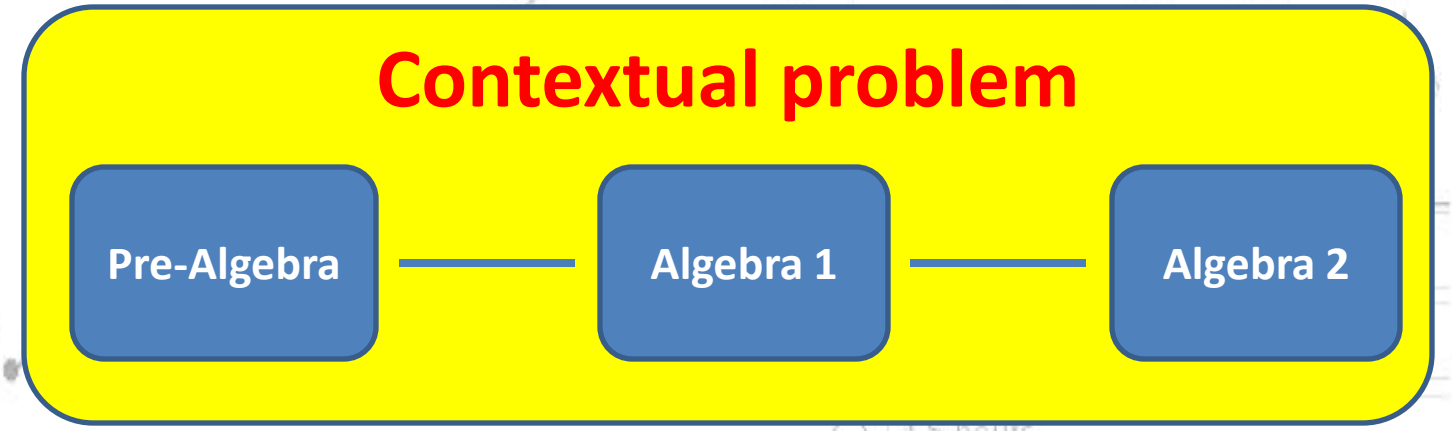
Lupe wants to earn money in the summer by mowing lawns. The going rate is $7.50 per hour if Lupe uses the customer's lawn mower. How much money can Lupe earn in a summer?

This problem can be changed slightly. Suppose Lupe considers buying her own lawn mower for $250 and charging her customers $12 per hour. Which plan will give Lupe the most profit?

Phillips, Elizabeth. Patterns and Functions, Addenda Series Grades 5-8. National Council of Teachers of Mathematics (NCTM): Reston, VA, 1991.

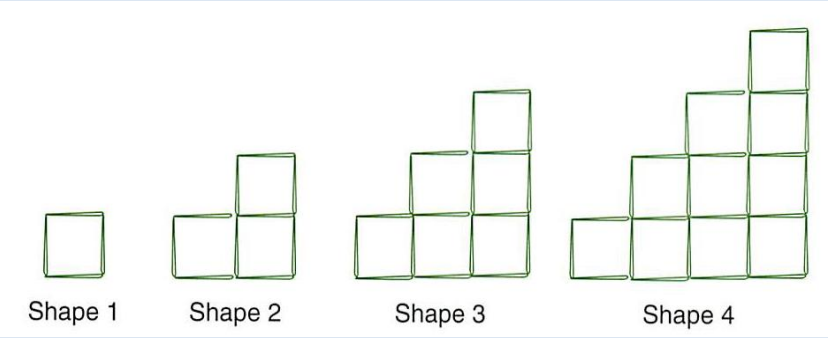
Problems with Access Points

We will look at some problems that have what I term "multiple access points." Teachers and students can enter at various points of the problem based on their background and readiness.



Pre-Algebra Access Point

Building with Toothpicks



1. Use a pattern from the shapes to determine the perimeter of the fifth shape in the sequence. Show or explain how you arrived at your answer.
2. Write a formula that you could use to find the perimeter of any shape *n*. Explain how you found your formula.

Friel, Susan, Sid Rachlin, and Dot Doyle. Navigating through Algebra in grades 6-8. National Council of Teachers of Mathematics (NCTM): Reston, VA, 2001.

Representation Standard

"It is important to encourage students to represent their ideas in ways that make sense to them, even if their first representations are not conventional ones. It is also important that they learn conventional forms of representation to facilitate both their learning of mathematics and their communication with others about mathematical ideas"

_____. *Principles and Standards for School Mathematics.* National Council of Teachers of Mathematics (NCTM): Reston, VA. 2000.

What is Modeling?

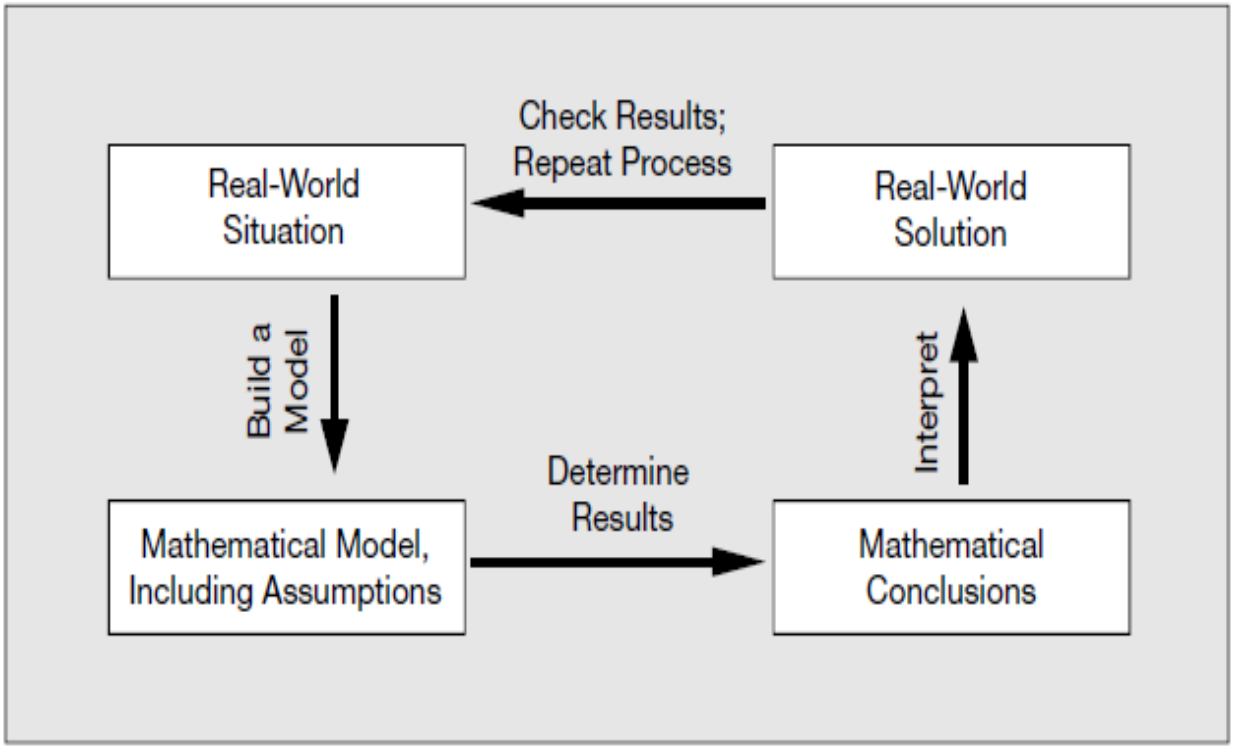
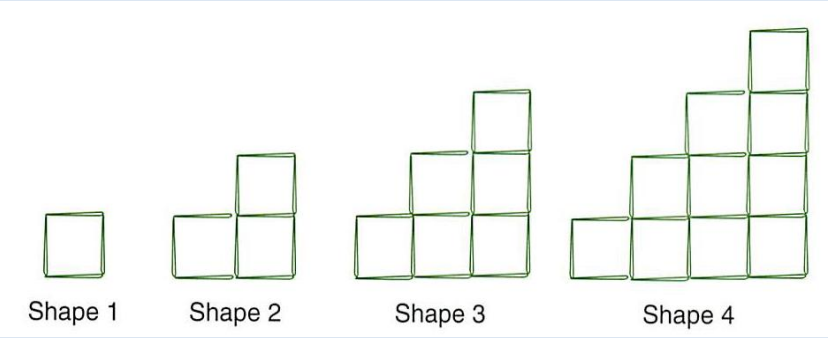


Fig. 2.2. Four-step modeling cycle used to organize reasoning about mathematical modeling

_____. Focus in High School Mathematics: Reasoning and Sense Making. National Council of Teachers of Mathematics (NCTM): Reston, VA, 2009.

Algebra 1 Access Point

Building with Toothpicks



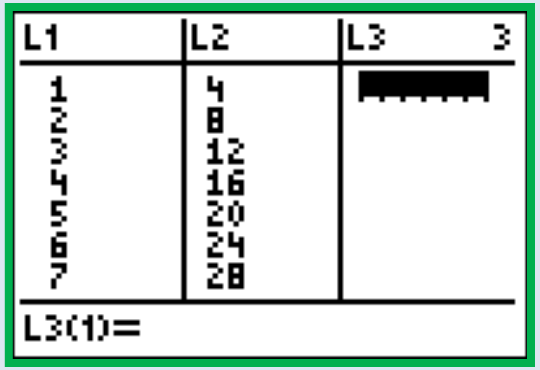
3. Create a table and a graph of the first seven shapes in the pattern. What rule did you use to continue the pattern? Explain how you determined your rule.

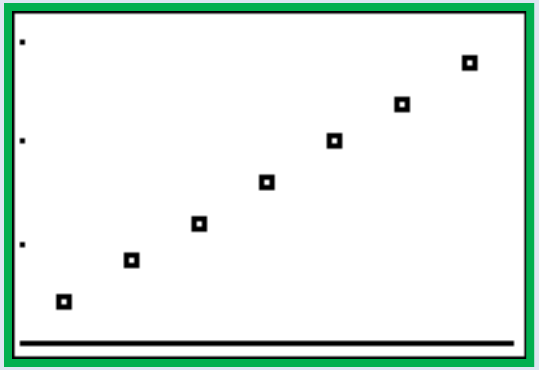
4. How would the pattern differ if you used triangles instead of squares?

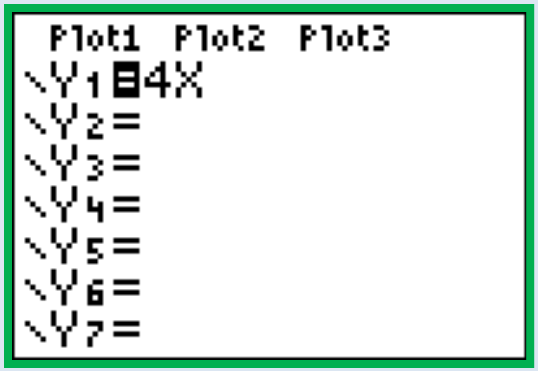
Extension from: Friel, Susan, Sid Rachlin, and Dot Doyle. Navigating through Algebra in grades 6-8. National Council of Teachers of Mathematics (NCTM): Reston, VA, 2001.

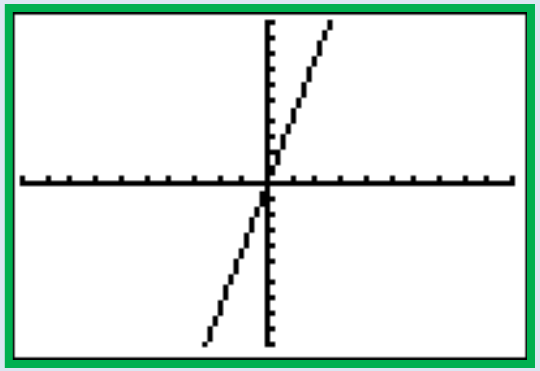
Algebra 1 Access Point

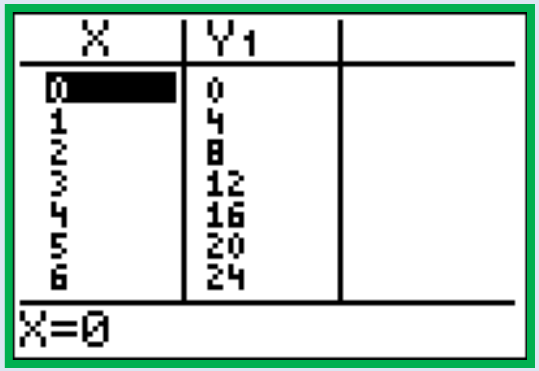
Building with Toothpicks



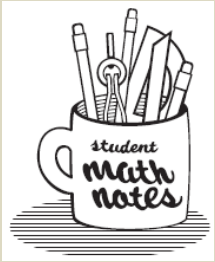


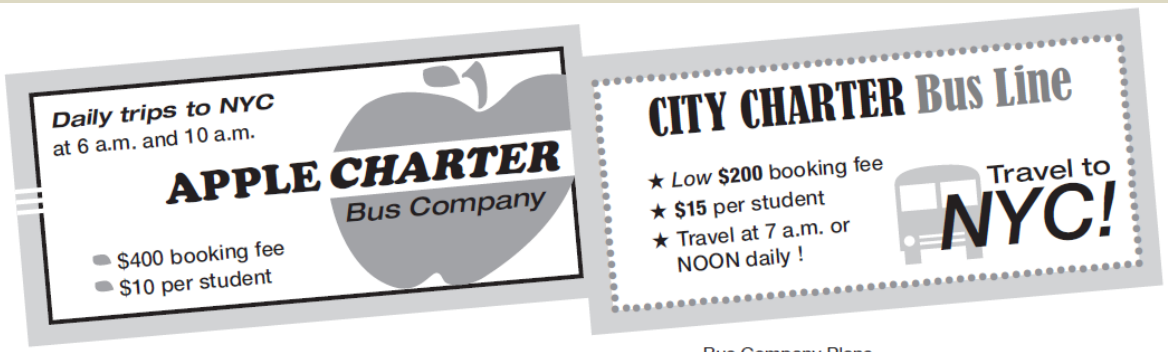






SMN's *Big Apple or Bust*





1. Which bus company plan do you think is the better deal? Explain why you think so.

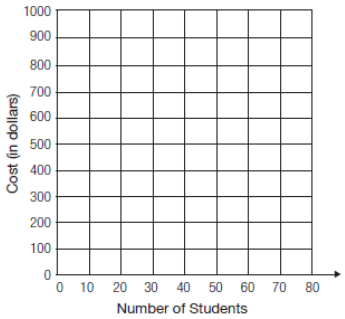
2. Complete the table below.

Cost Comparison

Number of Students	Apple Charter	City Charter
5		
15		
25		
35		
45		

3. Now use the table to describe which bus company you think is the better deal, and compare it to your answer in question 1.

Bus Company Plans



5. Describe in words how to find the cost of using Apple Charter Bus Company with any number of students:

The cost will equal _____ plus _____

_____

6. Use variables and numbers to write an equation that describes the cost of using Apple Charter Bus Company with any number of students. Let *C* represent the cost and *s* the number of students:

C = _____

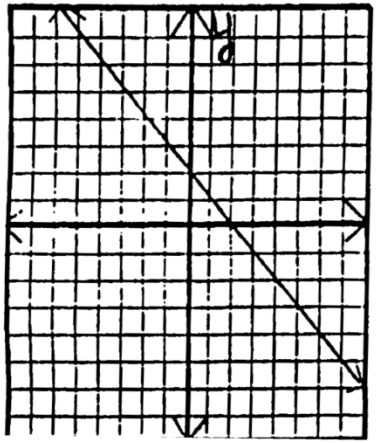
_____. "Big Apple or Bust," *Student Math Notes*, National Council of Teachers of Mathematics. May/June 2009.

Multiple Representations

- Verbal
- Concrete/Manipulatives
- Numerical
- Graphical
- Algebraic

What is the role of representations in instruction?

Jigsaw Activity



$y = -x + 2$

x	-4	-2	0	2
y	6	4	2	0

y is two more than the opposite of x.

Reading and Questioning

The need to read problems carefully is vital – and to learn the importance of re-reading

Determine what is given in the problem and what is being asked

Get students to "unpack" their thinking and ask themselves questions

Ask probing questions to encourage student thinking

Encourage student reflection on their problem solving process

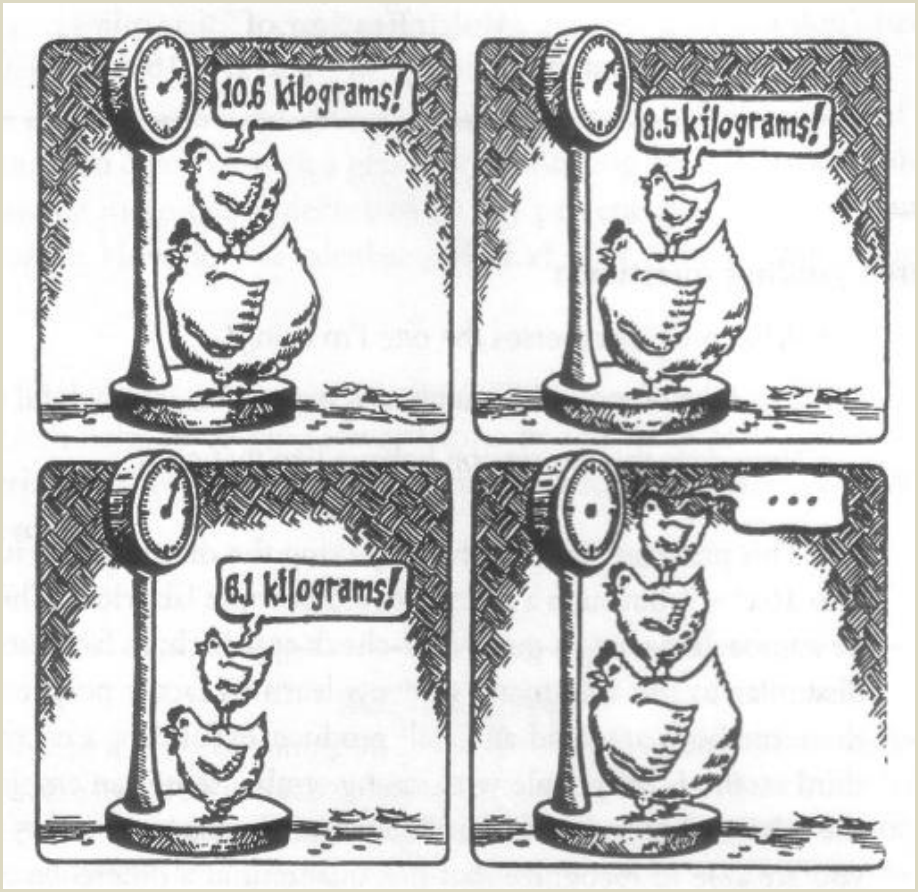
Multiple Representations Promote Learning

A store is having a sale and taking 35% off the price on all items. Create a table of input-output values, a graph, and an algebraic representation to represent this situation.

Moses, Barbara, editor. Algebraic Thinking Grades K-12. National Council of Teachers of Mathematics (NCTM): Reston, VA, 1999.
Van Dyke, Frances and Timothy V. Craine. "Equivalent Representations in the Learning of Algebra," *Mathematics Teacher*, November 1997.

“Chicken/Scale” Problem

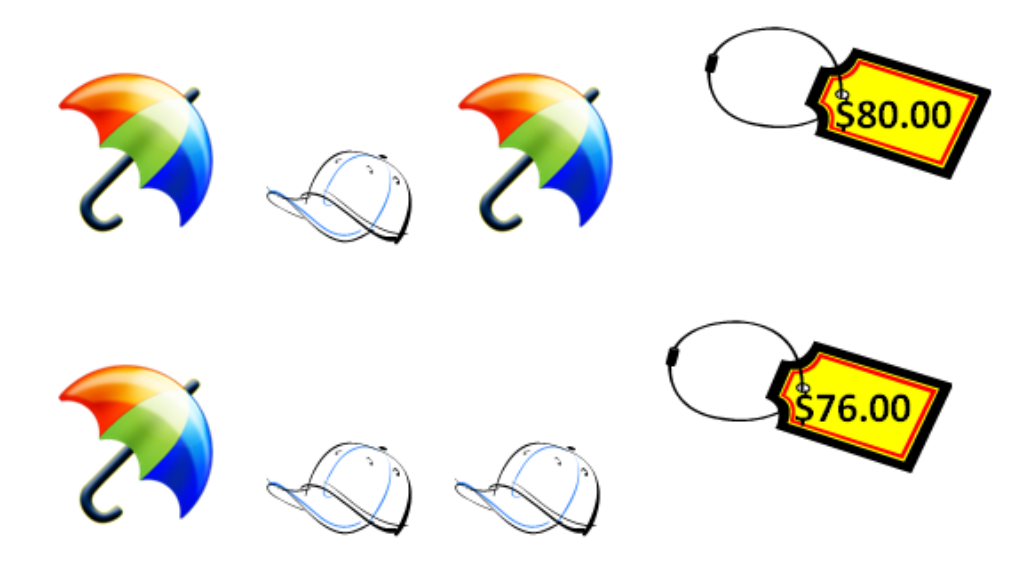
Three chickens were weighed in pairs; the first pair weighed in at 10.6 kg, the second pair weighed 8.5 kg, and the third pair weighed 6.1 kg. How much would the scale read if all three chickens were weighed at the same time? How many kilograms does each chicken weigh?



Driscoll, Mark. Fostering Algebraic Thinking, A Guide for Teachers Grades 6-10. Heinemann Publishing: Portsmouth, Hew Hampshire, 1999.

“Umbrella/Cap” Problem

Finding Prices. The following picture shows the costs of two combinations of umbrellas and hats:



What is the price of one umbrella? One cap?

Driscoll, Mark. Fostering Algebraic Thinking, A Guide for Teachers Grades 6-10. Heinemann Publishing: Portsmouth, Hew Hampshire, 1999.

"Umbrella/Cap" Problem

Combination chart solution:

Follow pattern, then solve

Costs of Combinations (in dollars)

Number of Umbrellas							
	5						
	4						
	3	*84*					
	2		80				
	1			76			
	0				*72*		
		0	1	2	3	4	5
		Number of Caps					

Cuoco, Albert A. and Frances R. Curcio, editors. The Roles of Representation in School Mathematics. National Council of Teachers of Mathematics (NCTM): Reston, VA, 2001.

Technology Principle

"When technological tools are available, students can focus on decision making, reflection, reasoning, and problem solving"

"Technology enriches the range and quality of investigations by providing a means of viewing mathematical ideas from multiple perspectives"

_____. *Principles and Standards for School Mathematics.* National Council of Teachers of Mathematics (NCTM): Reston, VA. 2000.

Example from TI-Nspire™

7.3 8.1 8.2 9.1 RAD AUTO REAL

Public Release Version 2008, #23

William charges $4 per hour to babysit. LaRhonda charges $10, plus an additional $2 per hour to babysit. Both William and LaRhonda work the same number of hours. After how many hours will they earn the same amount of money?

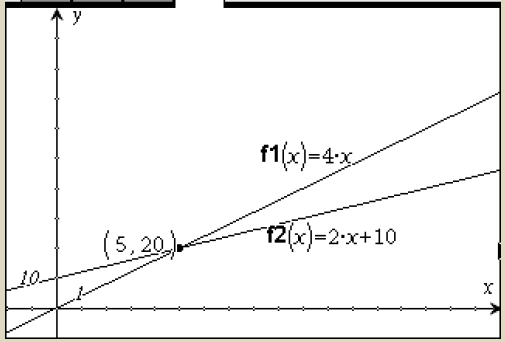
8.1 8.2 9.1 9.2 RAD AUTO REAL

This problem is similar to an earlier one. We can solve this problem using a graph to find the point of intersection graphically, creating two equations and finding their point of intersection algebraically, or creating a table of values for each job and comparing the amounts.

8.2 9.1 9.2 9.3 RAD AUTO REAL

First, let's find the graphic point of intersection. Graph the lines, create an appropriate window, then use MENU/ Points and Lines/Intersection Points to determine the coordinates of the point of intersection.

9.1 9.2 9.3 9.4 RAD AUTO REAL



9.2 9.3 9.4 9.5 RAD AUTO REAL

The second method would be to find the point of intersection of the two equations.

9.3 9.4 9.5 9.6 RAD AUTO REAL

solve(4·x=2·x+10,x)

0/99

9.4 9.5 9.6 9.7 RAD AUTO REAL

A third method would be to create a table of values for each of the equations and then look for when the two lists have a common value.

9.5 9.6 9.7 9.8 RAD AUTO REAL

	A a	B b	C	D
1	4	12		
2	8	14		
3	12	16		
4	16	18		
5	20	20		

A1 4

9.6 9.7 9.8 9.9 RAD AUTO REAL

After how many hours will they (William and LaRhonda) earn the same amount of money?

- ○ 2 hours
- ○ 2.5 hours
- ○ 4.5 hours
- ○ 5 hours

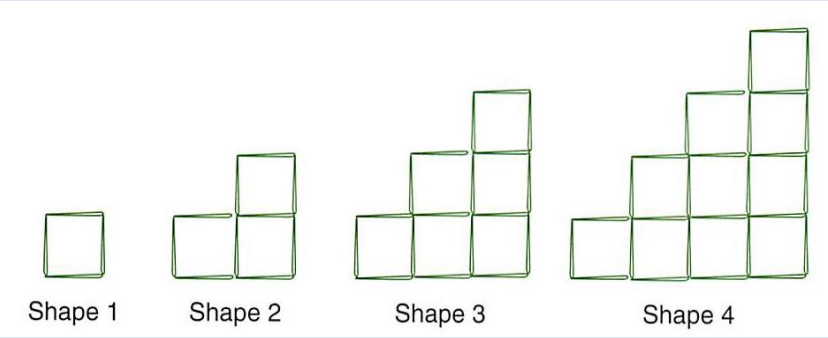
Representation Standard

"Students will be better able to solve a range of algebra problems if they can move easily from one type of representation to another . . . Students should also become flexible in recognizing equivalent forms of linear equations and expressions. This flexibility can emerge as students gain experience with multiple ways of representing a contextualized problem."

_____. *Principles and Standards for School Mathematics.* National Council of Teachers of Mathematics (NCTM): Reston, VA. 2000.

Algebra 2 Access Point

Building with Toothpicks



5. Determine the explicit and recursive formulas for finding the perimeter of the n^{th} figure.

6. What would be the perimeter of the 100^{th} figure?

Extension from: Friel, Susan, Sid Rachlin, and Dot Doyle. Navigating through Algebra in grades 6-8. National Council of Teachers of Mathematics (NCTM): Reston, VA, 2001.

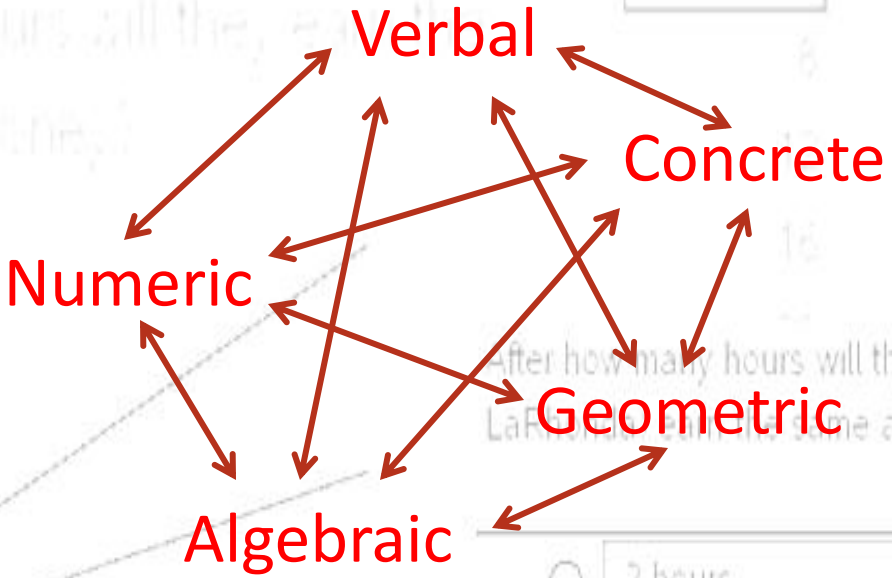
Strategy "Toolkit"

"Building fluency in working with algebraic notation that is grounded in reasoning and sense making will ensure that students can flexibly apply the powerful tools of algebra in a variety of contexts both within and outside mathematics."

_____. Focus in High School Mathematics: Reasoning and Sense Making. National Council of Teachers of Mathematics (NCTM): Reston, VA, 2009.

Generalizing

We need to assist our students in moving among representations:



to generalize strategies and solve problems.

Equity Principle

"Equity requires accommodating differences to help everyone learn mathematics"

" All students should have access to an excellent and equitable mathematics program that provides solid support for their learning and is responsive to their prior knowledge, intellectual strengths, and personal interests"

_____. *Principles and Standards for School Mathematics.* National Council of Teachers of Mathematics (NCTM): Reston, VA. 2000.

Additional Problems to Consider

- The Devil and Daniel Webster (payment options)
- Job-Offer Options (commission)
- Cell phone plans
- Medicine absorption
- Growth Rates (Linear and Exponential)

How Does Your Pattern Grow?

Student Explorations in Mathematics

Formerly *Student Math Notes*

Adding It Up

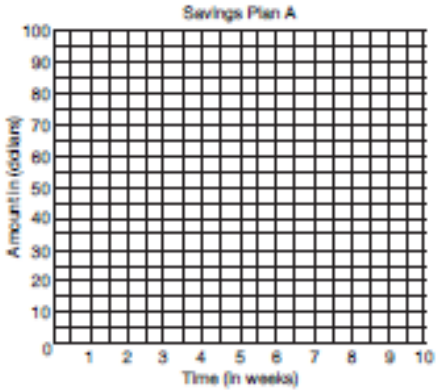
Imagine that you want to purchase the new *Larry Kotter and the Knights of Pythgoras* book on CD, which costs $49.95. You have only $25.50 right now. Your allowance is $5 per week.

1. First, guess how many weeks it will take you to save enough money for the CD.

2. Now use the table to determine how many weeks it will take you to save for the CD.

Week	Add	Total
0	–	$25.50
1	$5.00	$30.50
2		
3		
4		
5		
6		
7		
8		
9		

3. Using the table above and the grid in the next column, graph the information from both the number of weeks and the amount of money saved.



4. Should you connect the dots on your graph? Why, or why not?

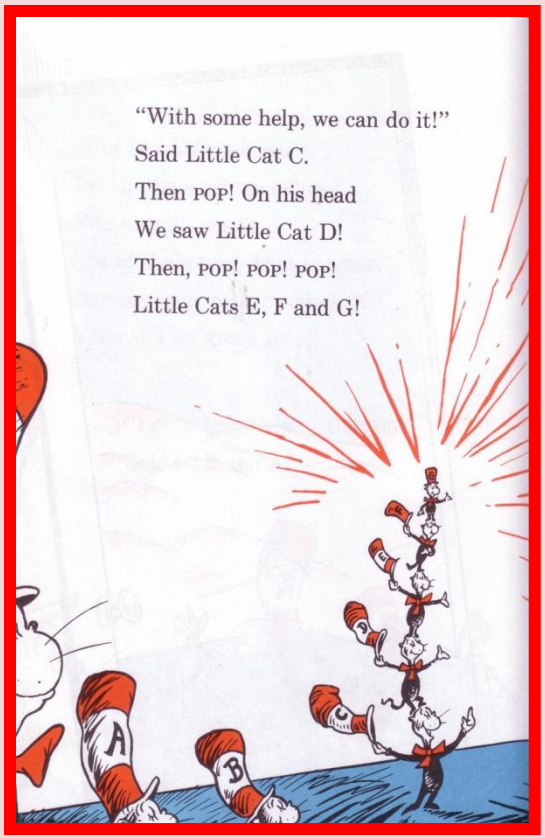
5. What does point (8, 65.5) on the graph represent in this situation?

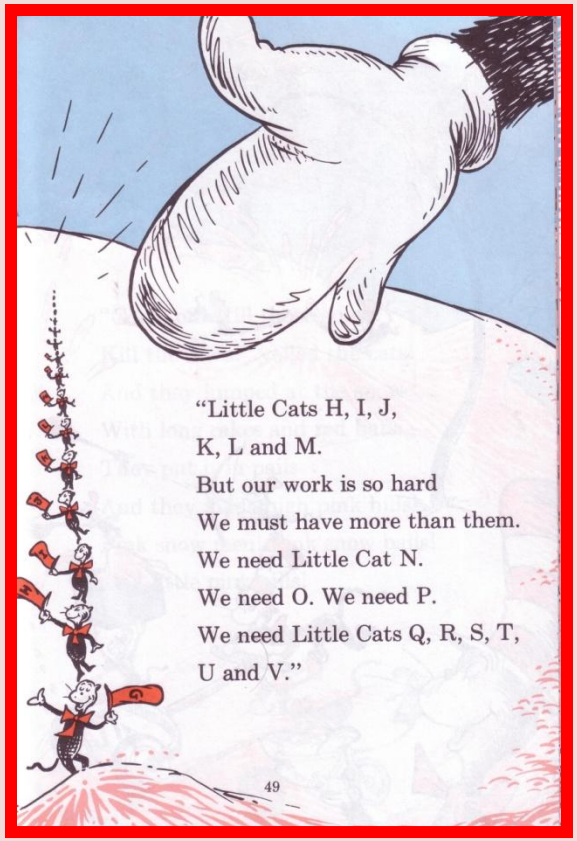
6. Use the table and the graph to determine how much money you will have at 10 weeks. Did you prefer using the table or the graph? Why?

This type of growth pattern in numbers—where you add a constant to each entry to obtain the next number—is called an **arithmetic sequence**. The pattern resembles a linear function but is not continuous.

_____. "How Does Your Pattern Grow?," *Student Explorations in Mathematics*, National Council of Teachers of Mathematics. September 2009.

The Cat in the Hat Comes Back





Geisel, Theodore Seuss. *The Cat in the Hat Comes Back,* Random House Beginner Books: Toronto, Canada, 1958.

Getting Your Reps for a Great Algebra Workout

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